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Engineering Notes

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Deorbit Process Using Solar Radiation Force

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Nomenclature

a	=	semimajor axis
E	=	energy
$\boldsymbol{e}$	=	eccentricity vector
F	=	solar radiation force on whole spacecraft per unit mass
F'	=	solar radiation force on solar panels per unit mass
$\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$	=	Cartesian unit vectors
$\boldsymbol{M}$	=	angular momentum vector per unit mass
n	=	mean motion
P	=	satellite orbit period
r_a	=	apogee radius
r_p	=	perigee radius
r_{syn}	=	geosynchronous radius
$\hat{\boldsymbol{r}}$	=	satellite position unit vector
$\hat{\boldsymbol{s}}$	=	sun unit vector
$\boldsymbol{v}$	=	instantaneous velocity vector
v_{syn}	=	geosynchronous velocity
α	=	sun right ascension
δ	=	sun declination
ε	=	inclination of ecliptic plane
θ	=	angular position from x axis
λ	=	sun mean longitude
μ	=	Earth gravitational constant, 398601.16 km^3/s^2

Introduction

THE growing satellite population in geostationary orbit makes it necessary to deorbit satellites at end of life to greater altitudes,[1–3] otherwise, they pose an increased risk of collision for current and future missions. The purpose of this Note is to demonstrate the feasibility of using the solar radiation force as a propulsion source to assist the deorbit process in three-axis stabilized satellites.

The solar radiation force effect on the semimajor axis for a satellite moving in geostationary orbit with panels tracking the sun occurs as a daily oscillation. When the satellite moves along its orbit away from the sun, the semimajor axis increases, and when the satellite moves toward the sun, the semimajor axis decreases, turning into a zero net change on one full orbit.

Therefore, by means of the timely rotation of the solar panels, it is possible to obtain a net change in the semimajor axis and an effect on the eccentricity, such that the perigee altitude increases secularly, helping to reduce the risk of collision with others satellites in geostationary orbit.

This method of increasing the orbit size can be very useful when the propulsion subsystem fails but it is still possible to drive its solar panels. This method also applies when the satellite does not have enough propellant to complete the deorbit process or when we only want to further increase the deorbit altitude provided by the standard process.[4,5] In this process, the satellite's propulsion is used to increase alternatively the opposite apsides of the orbit, taking into account the uncertainty of the remaining propellant to avoid a larger eccentricity in case of an early propellant depletion.

Although a lot of work has been done regarding the perturbations due to the solar radiation force on geosynchronous satellites,[6] our analysis relies on a simple model for the solar radiation force that depends on the sun declination but not on the time of the day. That is, we are averaging the solar radiation force during a day, but this average changes from day to day due to the yearly sun motion.

We consider only the effects of the solar radiation force in the semimajor axis and eccentricity vector because the perturbations in the other orbital elements are very small compared with those produced by gravitational forces.[7]

To implement this method to assist the deorbit process, it is necessary to maintain continuous ground control of the satellite. Alternatively, the onboard computer may be programmed to command autonomously the panels at the appropriate times. The attitude control subsystem should be able to manage the new configuration, and the batteries should provide the necessary energy when the panels are rotated. During the first part of orbit, the solar cells generate the power required for satellite operation, including the recharge of batteries. In the second half of the orbit, the solar cells are not illuminated by the sun; therefore, the power required should be provided by the batteries.

Solar Radiation Force Effect on Semimajor Axis

Figure 1 shows the orbit of a geostationary satellite and two reference frames. The first frame is the right ascension–declination with its origin in the center of the Earth. The XY plane is an equatorial plane extension. The X axis points toward Aries and the Z axis points toward the north pole. The second reference frame is superposed over the first one but is rotated around the Z axis such that the x axis points toward the projection of the line of sight of the sun over the XY plane. The angle ε (23.4 deg) between the ecliptic plane and the Earth's equator is also indicated.

The line of sight of the sun can be modeled by[7]

$$\hat{\boldsymbol{s}} = (\cos\lambda, \sin\lambda\cos\varepsilon, \sin\lambda\sin\varepsilon) \tag{1}$$

We first consider the satellite motion during a time interval less than or equal to an orbital period, and for this interval we will consider the sun fixed. We assume that the solar panels are perpendicular to the equator, and that the attitude control of the satellite is able to maintain this configuration.

In the two-body problem, the energy depends only on the semimajor axis

$$E = -(\mu/2a) \tag{2}$$

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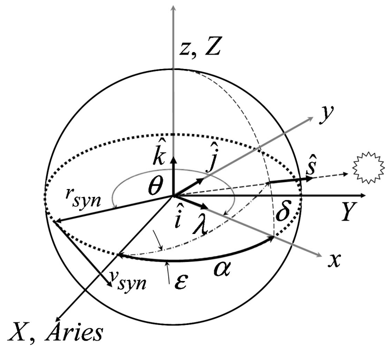

Fig. 1 Geostationary satellite reference frames.

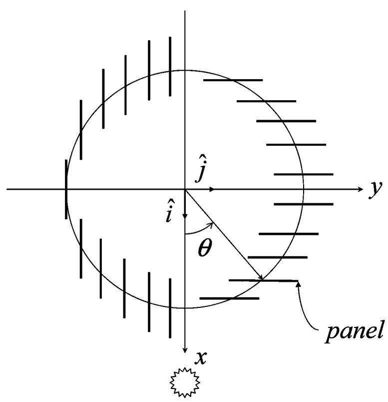

Fig. 2 Suggested positions of solar panels.

Differentiating Eq. (2), we obtain the change in semimajor axis when the energy changes

$$\frac{\mathrm{d}a}{a^2} = \frac{2}{\mu}\,\mathrm{d}E \tag{3}$$

For a geostationary orbit, and using Fig. 1, we rewrote Eq. (3) as

$$\frac{\mathrm{d}a}{a^2} = \frac{v_{\mathrm{syn}} PF \cos\delta}{\pi\mu}\sin\theta\,\mathrm{d}\theta \tag{4}$$

Integrating Eq. (4) over an interval less than or equal to one day, and assuming that the sun position is fixed for this time interval, we obtain the following change in the semimajor axis:

$$\Delta a = [(r_{\mathrm{syn}} PF\cos\delta)/(v_{\mathrm{syn}}\pi)](1-\cos\theta) \tag{5}$$

Notice that the semimajor axis increases for $0 < \theta < 180$ deg, reaching its maximum value at $\theta = 180$ deg, and decreases for $180 < \theta < 360$ deg, as expected. This is because the applied velocity increment $\Delta\boldsymbol{v}$ goes in the same direction as v_{syn} for $0 < \theta < 180$ deg and in the opposite direction for $180 < \theta < 360$ deg.

Given a typical three-axis stabilized satellite shape, with movement in orbit with its solar panels always facing the sun and its communication antennas pointing toward the Earth, we have a net semimajor axis change of approximately zero. We say approximately zero because Eq. (5) is valid only for a constant solar radiation force, and in our case the main variation of solar radiation force comes from asymmetry of the satellite body shape. The antennas do not have the same size, and they are not equally oriented with respect to the sun. However, we can assume an approximate symmetry around the z axis, so that for an angle θ such that $0 < \theta < 180$ deg there is a corresponding angle θ $180 < \theta < 360$ deg with approximately the same solar radiation force.

If we could turn off the solar radiation force just after the semimajor axis maximum is reached, we would have a net semimajor axis change. This will be partially accomplished if the solar panels are rotated 90 deg around the z axis, (Fig. 2).

To evaluate the net change in the semimajor axis, we decompose the solar radiation force per unit mass into two parts, one coming from the illumination of the solar panels (F') and the other coming from the illumination of the rest of the satellite ($F - F'$). Because

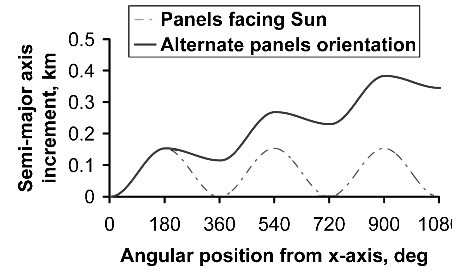

Fig. 3 Semimajor axis increment.

we have assumed symmetry around the z axis, the net change in a full orbit from $(F - F')$ is zero. When the solar panels are moved as shown in Fig. 2, the net change in the semimajor axis per orbit will be

$$\Delta a_{\mathrm{orbit}} = \frac{2 r_{\mathrm{syn}} PF' \cos\delta}{v_{\mathrm{syn}}\pi} \tag{6}$$

Once we have obtained the equation for the change of the semimajor axis per orbit, we must analyze what happens during a year. In this case, we have to integrate Eq. (6) again to include the yearly sun motion.

Notice that when the satellite completes a full orbit, the sun's right ascension changes approximately by

$$\Delta\alpha_{\mathrm{day}} = 2\pi/365.25 \tag{7}$$

Thus, integrating Eq. (6) over α for a full year, we get

$$\Delta a_{\mathrm{year}} = \frac{2 r_{\mathrm{syn}} PF'[2\pi - (1-\cos\varepsilon)\pi]}{v_{\mathrm{syn}}\pi}\left(\frac{365.25}{2\pi}\right) \tag{8}$$

the approximation $\lambda = \alpha$ was used to perform the integration (Fig. 1).

Assume a solar radiation force of 0.00026 N (solar panels plus spacecraft body plus antennas). This is the estimated solar radiation force using the range residuals in the spring equinox for a satellite with an end-of-life mass of 1280 kg.

Assuming that 75% of the solar radiation force is concentrated in the solar panels, and the remaining 25% in the communication antennas and spacecraft body, we can apply Eqs. (6) and (8) to a typical satellite at end of life to obtain 0.110 km as the average change in the semimajor axis per day and 40.23 km as the annual change.

Figure 3 shows the semimajor axis increment behavior over three orbital periods with panels always facing the sun and with panels being oriented alternatively. Figure 3 shows two curves and starts at the spring equinox when the solar radiation force over the satellite has its maximum value (0.00026 N). In both curves, when the satellite moves along the orbit between perigee and apogee, the semimajor axis increment grows at the same rate and reaches a value of 0.153 km. However, in the case when the panel's orientation is alternated and the satellite moves along the orbit between apogee and perigee, the total semimajor axis increment decreases to 0.114 km after one full orbit. This value can be compared with the yearly average semimajor increment (0.110 km).

Solar Radiation Force Effect on Eccentricity Vector

In the two-body problem, the Runge–Lenz or Laplace vector (see Ref. 8) provides us with an equation for the eccentricity in terms of the instantaneous velocity

$$\boldsymbol{e} = [(\boldsymbol{v}\times\boldsymbol{M})/\mu] - \hat{\boldsymbol{r}} \tag{9}$$

Because the satellite position does not change instantaneously by the $\Delta\boldsymbol{v}$ applied, the change in eccentricity when we start with a circular orbit is

$$\Delta\boldsymbol{e} = \{[(\boldsymbol{v}+\Delta\boldsymbol{v})\times(\boldsymbol{M}+\Delta\boldsymbol{M})]/\mu\} - \hat{\boldsymbol{r}} \tag{10}$$

which can be rewritten, after eliminating second-order terms, as

$$\Delta\boldsymbol{e} = \hat{\boldsymbol{i}}\left(\frac{\Delta v}{v_{\mathrm{syn}}}\sin\theta\cos\theta\right) + \hat{\boldsymbol{j}}\left(\frac{\Delta v}{v_{\mathrm{syn}}}\sin^2\theta + \frac{\Delta v}{v_{\mathrm{syn}}}\right) \tag{11}$$

Thus, for a satellite moving in a geostationary orbit with the panels pointing toward the sun, the change in eccentricity per day due to the solar radiation force is [7,9]

$$\Delta \boldsymbol{e}_{\text{day}} = \hat{\boldsymbol{j}}[(3P/2)(F \cos\delta / v_{\text{syn}})] \qquad (12)$$

that is, the change of the eccentricity vector leads the sun right ascension by 90 deg.

On the other hand, if we rotate the solar panels, the change in the eccentricity per day will be

$$\Delta \boldsymbol{e}_{\text{day}} = \hat{\boldsymbol{j}}[(3P/4v_{\text{syn}})F \cos\delta + (3P/4v_{\text{syn}})(F - F')\cos\delta] \qquad (13)$$

which is smaller than that obtained with the panels pointing toward the sun. [Refer to Eq. (12).] Nevertheless, in both cases, the solar radiation force effect will be a rotation of the eccentricity vector, but with different magnitude.

Using Eqs. (12) and (13), we obtain the average eccentricity increment per orbit as $8.21E$-6 for a satellite with its panels facing the sun, and $5.13E$-6 for a satellite with rotated panels. These numbers are obtained using the yearly average solar radiation force, 0.000249 N.

The time evolution of the eccentricity vector depends on the initial eccentricity vector. Thus, for a satellite with its eccentricity vector pointing approximately to the sun, the eccentricity vector experiences a small rotation. If we use Eq. (7) and choose the initial eccentricity vector magnitude $e_i = e_s$ such that

$$e_s = (365.25/2\pi)\Delta \boldsymbol{e}_{\text{day}} \qquad (14)$$

the eccentricity vector will follow the sun and will remain with approximately the same magnitude. We say approximately because we have neglected the solar and moon gravitational perturbations, and we have assumed a constant solar radiation force.[7,9]

In this case, the perigee and apogee radius increment will be

$$\Delta r_p = \Delta a(1 - e_s) > 0 \qquad (15)$$

$$\Delta r_a = \Delta a(1 + e_s) > 0 \qquad (16)$$

Equations (15) and (16) indicate that the perigee and apogee will increase monotonically. However, the apogee will increase a little faster than the perigee.

When the initial eccentricity vector magnitude is different than e_s, the path followed by the eccentricity vector will be a circle but will not be centered at the origin. The eccentricity magnitude will oscillate between e_i and $2e_s - e_i$, depending on the epoch of the year.[7,9] In this case, the perigee and apogee radii will continue increasing, but the rate will depend on the epoch of the year.

Conclusions

The solar radiation force can be used as a propulsion source to assist in the deorbit process in three-axis stabilized satellites with the capability to move their solar panels. This is achieved by rotating the solar panels at appropriate times to achieve an effective increment in the semimajor axis during a full orbit.

For a satellite with a total solar radiation force of 0.00026 N, a weight of 1280 kg, and assuming that 75% of the solar radiation force is concentrated on the solar panels, an average increment in the semimajor axis per day of 0.110 km and an annual increment of 40.23 km can be obtained. The growth rate of orbit perigee altitude will be constant and nearly the same as the growth rate of the semimajor axis when the initial eccentricity vector is properly chosen.

The Inter-Agency Debris Coordination Committee recommends a disposal orbit with a perigee altitude higher than 235 km above the ideal geosynchronous radius of 42,164 km. At a perigee altitude growth rate of 40.23 km/year, it will take the satellite more than 5.8 years to reach the recommended orbit.

Hence, this method would also be used as an alternative to assist in the deorbit process of three-axis stabilized satellites when the propulsion subsystem fails, provided that they can still be commanded and that they still have solar panel drive capability, or when there is not enough propellant to complete the deorbit process. Because many satellites have been unable to perform end-of-life disposal due to propulsion system difficulties, this method may improve the overall success rate of geosynchronous satellite disposal.

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